

Mass splittings of the baryon decuplet and antidecuplet with the second-order flavor symmetry breakings within a chiral soliton model

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We revisit the mass splittings of SU(3) baryons, taking into account the second-order effects of isospin and SU(3) flavor symmetry breakings within the framework of a chiral soliton model. The masses of the baryon decuplet turn out to be improved, compared to those with the first-order corrections. The mass of the N^* as a member of the baryon antidecuplet is obtained as $M_{n^*} = 1687$ MeV, which is in agreement with the recent experimental data. The pion-nucleon sigma term becomes $\Sigma_{\pi N} = (50.5 \pm 5.4)$ MeV.

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I. INTRODUCTION

The Θ^+ baryon [1–3], which is the first excited exotic pentaquark state, has drawn much attention, since the LEPS collaboration announced the evidence of its existence [4]. However, the null results of the CLAS experiments about Θ^+ [5–8], have cast doubt upon its existence. In the meanwhile, the DIANA collaboration published the positive evidence of Θ^+ [9, 10]. Very recently, it has announced the formation of a narrow pK^0 peak with mass of 1538 ± 2 MeV/ c^2 and width of $\Gamma = 0.39 \pm 0.10$ MeV in the $K^+n \rightarrow K^0p$ reaction with higher statistical significance ($6\sigma - 8\sigma$) [10]. In addition, other new positive experiments for the Θ^+ have been reported [11–14]. The LEPS collaboration reported again the evidence of the Θ^+ existence [15] with the data $M_\Theta = 1.524 \pm 0.002 \pm 0.003$ GeV/ c^2 given in the statistical significance 5.1σ .

In addition to Θ^+ , Kuznetsov et al. [16–18] discovered a new nucleon-like resonance around 1.67 GeV from η photoproduction off the deuteron in the neutron channel, based on the GRAAL data. The decay width was measured to be around 40 MeV. Excluding the effects of the Fermi-motion, Fix et al. [19] argued that the width would further decrease. A important point is that this resonant structure is only seen in the neutron channel, which is the typical characteristics for the photo-excitation of the non-strange antidecuplet pentaquark [20, 21]. Very recently, a new analysis of the free proton GRAAL data [22–26] has revealed a resonance structure with a mass around 1685 MeV and width $\Gamma \leq 15$ MeV. The CB-ELSA collaboration [27] has confirmed an evidence for this N^* resonance. All these experimental facts are consistent with the results for the transition magnetic moments in the χ QSM [20, 21] and phenomenological analysis for the non-strange pentaquark baryons [28]. Based on these results, theoretical calculations of the $\gamma N \rightarrow \eta N$ reaction [29, 30] were shown to describe qualitatively well the GRAAL data. In Refs. [31–33] the non-strange partners of the Θ^+ were also studied.

In the present dubious situation related to the existence of Θ^+ , one needs to carefully review the previous theoretical analyses [2, 34] of the mass splittings of SU(3) baryons. The original analysis [2] was partially based on specific model calculations [35, 36], while some dynamical parameters are fixed by some experimental masses of the baryon octet and decuplet, and the empirical value of the πN sigma term $\Sigma_{\pi N}$. Moreover, Diakonov et al. [2] assumed then $N^*(1710)$ to be a member of the antidecuplet. Furthermore, the second moment of inertia is an essential quantity in determining the shift of the antidecuplet center from the octet center in the chiral limit but is known only in a wide range: $0.43 \text{ fm} < I_2 < 0.55 \text{ fm}$, depending on specific models such as either the Skyrme model [37, 38] or the chiral quark-soliton model (χ QSM) [39, 40]. Thus, some of model-dependent uncertainties are inexorable in previous analyses of the SU(3) baryon masses.

While the formalism of the baryon mass splittings was well established within chiral soliton models, the numerical analyses are still incomplete, because not all parameters can be fixed unequivocally, as mentioned already. In order

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to avoid this ambiguity, it is essential to consider the breakdown of isospin symmetry, without which it is simply not possible to take as input the experimental data of the octet baryon masses. The effects of isospin symmetry breaking consist of two different contributions: The electromagnetic (EM) and hadronic ones. Those from the EM corrections were already investigated in Ref. [41] within the same framework of the χ SM. Together with these EM corrections, the present authors carried out a new analysis of the mass splittings of the SU(3) baryons [42]. Distinguished from the previous works [2, 34], all the dynamical parameters were determined unambiguously, based on the experimental baryon octet, Ω^- and Θ^+ masses. We also showed that the width of Θ^+ and the transition magnetic moments for $N^*(1685) \rightarrow N\gamma$ turned out to be consistent with the analysis of the baryon mass splittings [43]. Moreover, the πN sigma term, $\Sigma_{\pi N}$, was predicted to be $\Sigma_{\pi N} = (36.4 \pm 3.9)$ MeV. In the present work, we want to extend the investigation, taking into account the second-order corrections of isospin and SU(3) symmetry breakings. It is important to examine how stable the results with the first-order effects of SU(3) and isospin symmetry breaking and how much we can improve the numerical results in comparison with the existing experimental data. For example, it will be shown that including the second-order contributions leads to more consistent mass relations such as the Moriguchi mass formula [44].

The present work is sketched as follows: In Section II, we recapitulate all the formulae relevant to the mass splittings of the SU(3) baryons. In Section III, We also discuss the results with the second-order corrections. In the last Section, we summarize the present work and draw conclusions.

II. MASS SPLITTINGS OF THE SU(3) BARYONS FROM THE CHIRAL SOLITON MODEL

The formalism for the mass splittings of the SU(3) baryons is well known within chiral soliton models. In particular, the collective Hamiltonian of chiral solitons have been investigated within various versions of the χ SM such as the Skyrme model [45], chiral quark-soliton model [39, 40], and chiral hyperbag model[46]. We will recapitulate the relevant formulae necessary for discussion of the baryon mass splittings with the second-order SU(3) and isospin symmetry breakings. The collective Hamiltonian in the SU(3) χ SM is expressed as

$$H = \frac{1}{2I_1} \sum_{i=1}^3 \hat{J}_i^2 + \frac{1}{2I_2} \sum_{p=4}^7 \hat{J}_p^2 + m_{\text{ibr}} \left(\frac{\sqrt{3}}{2} \alpha D_{38}^{(8)}(A) + \beta \hat{T}_3 + \frac{1}{2} \gamma \sum_{i=1}^3 D_{3i}^{(8)}(A) \hat{J}_i \right) + m_{\text{sbr}} \left(\alpha D_{88}^{(8)}(A) + \beta \hat{Y} + \frac{1}{\sqrt{3}} \gamma \sum_{i=1}^3 D_{8i}^{(8)}(A) \hat{J}_i \right) + (m_u + m_d + m_s) \sigma, \quad (1)$$

where $I_{1(2)}$ are the soliton moments of inertia that are dependent on dynamics of specific formulations of the χ SM. The $\hat{J}_{i(p)}$ stand for the generators of the SU(3) group. The \hat{Y} and \hat{T}_3 denote the operators of the hypercharge and isospin third component, respectively. The m_u , m_d , and m_s represent the up, down, and strange current quark masses, respectively. \bar{m} , m_{ibr} , and m_{sbr} are defined respectively as

$$\bar{m} = \frac{m_u + m_d}{2}, \quad m_{\text{ibr}} = m_d - m_u, \quad m_{\text{sbr}} = m_s - \bar{m}. \quad (2)$$

The $D_{ab}^{(\mathcal{R})}(A)$ are the SU(3) Wigner D functions. The α , β , and γ encode dynamics of specific chiral soliton models. For example, in the chiral quark-soliton model, they are written as

$$\alpha = -\sigma + \frac{K_2}{I_2}, \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2 \left(\frac{K_1}{I_1} - \frac{K_2}{I_2} \right), \quad \sigma = -(\alpha + \beta) = \frac{1}{3} \frac{\Sigma_{\pi N}}{\bar{m}}. \quad (3)$$

The eigenstates of the rotational part of the collective Hamiltonian, i.e., of that without the symmetry breaking parts, are expressed in terms of the SU(3) Wigner D functions in representation \mathcal{R} :

$$\Psi_{(\mathcal{R}^*; Y' J T_3)}^{(\mathcal{R}; Y T T_3)}(A) = \sqrt{\dim(\mathcal{R})} (-)^{J_3 + Y'/2} D_{(Y, T, T_3)(-Y', J, -J_3)}^{(\mathcal{R})*}(A), \quad (4)$$

where \mathcal{R} denotes the corresponding representation of the SU(3) group, i.e., one of $\mathcal{R} = 8, 10, \overline{10}, \dots$. Y, T, T_3 are the corresponding hypercharge, isospin, and its third component, respectively. The constraint of the right hypercharge $Y' = 1$ determines a tower of allowed SU(3) representations: The lowest ones, that is, the baryon octet and decuplet, coincide with those of the quark model. It provides a certain duality between rigidly rotating heavy soliton and constituent quark model. The third lowest representation is the antidecuplet [2].

If one turns on SU(3) symmetry breaking, the collective wave functions starts to get mixed with other representations[47]

$$|B_8\rangle = |8_{1/2}, B\rangle + c_{10}^B |\overline{10}_{1/2}, B\rangle + c_{27}^B |27_{1/2}, B\rangle,$$

$$\begin{aligned}
|B_{10}\rangle &= |10_{3/2}, B\rangle + a_{27}^B |27_{3/2}, B\rangle + a_{35}^B |35_{3/2}, B\rangle, \\
|B_{\overline{10}}\rangle &= |\overline{10}_{1/2}, B\rangle + d_8^B |8_{1/2}, B\rangle + d_{27}^B |27_{1/2}, B\rangle + d_{\overline{35}}^B |\overline{35}_{1/2}, B\rangle.
\end{aligned} \tag{5}$$

Here, the spin indices J_3 have been suppressed. The mixing coefficients in Eq.(5) read as follows

$$\begin{aligned}
c_{\overline{10}}^B &= c_{\overline{10}} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad c_{27}^B = c_{27} \begin{bmatrix} \sqrt{6} \\ 3 \\ 2 \end{bmatrix}, \quad a_{27}^B = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{bmatrix}, \quad a_{35}^B = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{bmatrix}, \\
d_8^B &= d_8 \begin{bmatrix} 0 \\ \sqrt{5} \\ \sqrt{5} \\ 0 \end{bmatrix}, \quad d_{27}^B = d_{27} \begin{bmatrix} 0 \\ \sqrt{3/10} \\ 2/\sqrt{5} \\ \sqrt{3/2} \end{bmatrix}, \quad d_{\overline{35}}^B = d_{\overline{35}} \begin{bmatrix} 1/\sqrt{7} \\ 3/(2\sqrt{14}) \\ 1/\sqrt{7} \\ \sqrt{5/56} \end{bmatrix},
\end{aligned} \tag{6}$$

respectively in the bases $[N, \Lambda, \Sigma, \Xi], [\Delta, \Sigma^*, \Xi^*, \Omega], [\Theta^+, N_{\overline{10}}, \Sigma_{\overline{10}}, \Xi_{\overline{10}}]$ and states in $\mathcal{R} = 27, 35, \overline{35}$. The coefficients in the mixing parameters are written in terms of α and γ

$$\begin{aligned}
c_{\overline{10}} &= -\frac{I_2}{15} m_{\text{sbr}} \left(\alpha + \frac{1}{2} \gamma \right), \quad c_{27} = -\frac{I_2}{25} m_{\text{sbr}} \left(\alpha - \frac{1}{6} \gamma \right), \quad a_{27} = -\frac{I_2}{8} m_{\text{sbr}} \left(\alpha + \frac{5}{6} \gamma \right), \\
a_{35} &= -\frac{I_2}{24} m_{\text{sbr}} \left(\alpha - \frac{1}{2} \gamma \right), \quad d_8 = \frac{I_2}{15} m_{\text{sbr}} \left(\alpha + \frac{1}{2} \gamma \right), \quad d_{27} = -\frac{I_2}{8} m_{\text{sbr}} \left(\alpha - \frac{7}{6} \gamma \right), \\
d_{\overline{35}} &= -\frac{I_2}{4} m_{\text{sbr}} \left(\alpha + \frac{1}{6} \gamma \right).
\end{aligned} \tag{7}$$

Here, we collect the expressions for the masses of the baryon octet, decuplet, and antidecuplet. The first-order corrections of both isospin and SU(3) symmetry breakings can be obtained as

$$\begin{aligned}
M_N^{(1)} &= m_{\text{ibr}} \left(\frac{1}{10} \alpha + \beta - \frac{7}{20} \gamma \right) T_3 + m_{\text{sbr}} \left(\frac{3}{10} \alpha + \beta - \frac{1}{20} \gamma \right), \\
M_\Lambda^{(1)} &= m_{\text{sbr}} \left(\frac{1}{10} \alpha + \frac{3}{20} \gamma \right), \\
M_\Sigma^{(1)} &= m_{\text{ibr}} \left(\frac{1}{4} \alpha + \beta - \frac{1}{8} \gamma \right) T_3 - m_{\text{sbr}} \left(\frac{1}{10} \alpha + \frac{3}{20} \gamma \right), \\
M_\Xi^{(1)} &= m_{\text{ibr}} \left(\frac{2}{5} \alpha + \beta + \frac{1}{10} \gamma \right) T_3 - m_{\text{sbr}} \left(\frac{1}{5} \alpha + \beta - \frac{1}{5} \gamma \right), \\
M_\Delta^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right) T_3 + m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right), \\
M_{\Sigma^*}^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right) T_3, \\
M_{\Xi^*}^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right) T_3 - m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right), \\
M_{\Omega^-}^{(1)} &= -2m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{5}{16} \gamma \right), \\
M_{\Theta^+}^{(1)} &= 2m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right), \\
M_{N^*}^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right) T_3 + m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right), \\
M_{\Sigma_{\overline{10}}}^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right) T_3, \\
M_{\Xi_{3/2}}^{(1)} &= m_{\text{ibr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right) T_3 - m_{\text{sbr}} \left(\frac{1}{8} \alpha + \beta - \frac{1}{16} \gamma \right).
\end{aligned} \tag{8}$$

The second-order corrections to the masses of the SU(3) baryons can be derived perturbatively [34]:

$$M_{B_\nu}^{(2)} = - \sum_{n \neq \nu} \frac{|\langle B_n | H_{\text{sb}} | B_\nu \rangle|^2}{\Delta M_{n-\nu}^{(0)}}, \tag{9}$$

where $\Delta M_{n \neq \nu}^{(0)}$ denotes the differences between the centers of the multiplets [2]. The second-order corrections are given as

$$\begin{aligned}
M_N^{(2)} &= -I_2 T_3^2 \left[\frac{1}{750} m_{\text{ibr}}^2 \left(25 \left(\alpha - \frac{1}{6} \gamma \right)^2 + 2 \left(\alpha + \frac{1}{18} \gamma \right)^2 \right) + \frac{2}{375} m_{\text{sbr}}^2 \left(18 \left(\alpha - \frac{1}{6} \gamma \right)^2 + 25 \left(\alpha + \frac{1}{2} \gamma \right)^2 \right) \right], \\
M_\Lambda^{(2)} &= -\frac{9}{250} I_2 m_{\text{sbr}}^2 \left(\alpha - \frac{1}{6} \gamma \right)^2, \\
M_\Sigma^{(2)} &= -I_2 \left[\frac{1}{120} m_{\text{ibr}}^2 \left(\alpha - \frac{1}{6} \gamma \right)^2 T_3^2 + \frac{1}{750} m_{\text{sbr}}^2 \left(12 \left(\alpha - \frac{1}{6} \gamma \right)^2 + 25 \left(\alpha + \frac{1}{2} \gamma \right)^2 \right) \right], \\
M_\Xi^{(2)} &= -I_2 T_3^2 \left[m_{\text{ibr}}^2 \frac{1}{375} \left(\alpha + \frac{1}{18} \gamma \right)^2 + \frac{12}{125} m_{\text{sbr}}^2 \left(\alpha - \frac{1}{6} \gamma \right)^2 \right], \\
M_\Delta^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{1}{2688} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{5}{384} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) T_3^2 + m_{\text{sbr}}^2 \left(\frac{25}{2688} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{15}{128} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) \right], \\
M_{\Sigma^*}^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{5}{5376} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{9}{256} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) T_3^2 + m_{\text{sbr}}^2 \left(\frac{5}{336} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{1}{16} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) \right], \\
M_{\Xi^*}^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{5}{21504} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{49}{3072} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) + m_{\text{sbr}}^2 \left(\frac{5}{5376} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{49}{768} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) T_3^2 \right. \\
&\quad \left. + m_{\text{sbr}}^2 \left(\frac{15}{896} \left(\alpha - \frac{1}{2} \gamma \right)^2 + \frac{3}{128} \left(\alpha + \frac{5}{6} \gamma \right)^2 \right) \right], \\
M_{\Omega^-}^{(2)} &= -I_2 \frac{5}{336} (m_s - \bar{m})^2 \left(\alpha - \frac{1}{2} \gamma \right)^2, \\
M_{\Theta^+}^{(2)} &= -\frac{3}{112} I_2 m_{\text{sbr}}^2 \left(\alpha + \frac{1}{6} \gamma \right)^2, \\
M_{N^*}^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{3}{896} \left(\alpha - \frac{1}{18} \gamma \right)^2 - \frac{1}{30} \left(\alpha - \frac{1}{6} \gamma \right)^2 + \frac{49}{1920} \left(\alpha + \frac{7}{18} \gamma \right)^2 \right) T_3^2 \right. \\
&\quad \left. + m_{\text{sbr}}^2 \left(\frac{3}{640} \left(\alpha - \frac{7}{6} \gamma \right)^2 + \frac{27}{896} \left(\alpha + \frac{1}{6} \gamma \right)^2 - \frac{1}{30} \left(\alpha + \frac{1}{2} \gamma \right)^2 \right) \right], \\
M_{\Sigma_{\text{T0}}}^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{3}{1792} \left(\alpha - \frac{1}{18} \gamma \right)^2 - \frac{1}{120} \left(\alpha - \frac{1}{6} \gamma \right)^2 - \frac{9}{1280} \left(\alpha + \frac{7}{18} \gamma \right)^2 \right) T_3^2 \right. \\
&\quad \left. + m_{\text{sbr}}^2 \left(\frac{1}{80} \left(\alpha - \frac{7}{6} \gamma \right)^2 + \frac{3}{112} \left(\alpha + \frac{1}{6} \gamma \right)^2 - \frac{1}{30} \left(\alpha + \frac{1}{2} \gamma \right)^2 \right) \right], \\
M_{\Xi_{\text{T0}}}^{(2)} &= -I_2 \left[m_{\text{ibr}}^2 \left(\frac{3}{4480} \left(\alpha - \frac{1}{18} \gamma \right)^2 + \frac{1}{384} \left(\alpha + \frac{7}{18} \gamma \right)^2 \right) T_3^2 \right. \\
&\quad \left. + m_{\text{sbr}}^2 \left(\frac{3}{128} \left(\alpha - \frac{7}{6} \gamma \right)^2 + \frac{15}{896} \left(\alpha + \frac{1}{6} \gamma \right)^2 \right) \right]. \tag{10}
\end{aligned}$$

The masses of the SU(3) baryons can be expressed in terms of $M_B^{(1)}$ and $M_B^{(2)}$:

$$M_B = \overline{M}_B + M_B^{(1)} + M_B^{(2)}, \tag{11}$$

where \overline{M}_B stand for the center masses of the multiplets.

The EM mass corrections to SU(3) baryon masses were already discussed in Ref. [41]. We compile the corresponding formulae here for the baryon octet

$$\begin{aligned}
M_N^{\text{EM}} &= \frac{1}{5} \left(c^{(8)} + \frac{4}{9} c^{(27)} \right) T_3 + \frac{3}{5} \left(c^{(8)} + \frac{2}{27} c^{(27)} \right) \left(T_3^2 + \frac{1}{4} \right) + c^{(1)}, \\
M_\Lambda^{\text{EM}} &= \frac{1}{10} \left(c^{(8)} - \frac{2}{3} c^{(27)} \right) + c^{(1)}, \\
M_\Sigma^{\text{EM}} &= \frac{1}{2} c^{(8)} T_3 + \frac{2}{9} c^{(27)} T_3^2 - \frac{1}{10} \left(c^{(8)} + \frac{14}{9} c^{(27)} \right) + c^{(1)},
\end{aligned}$$

$$M_{\Xi}^{\text{EM}} = \frac{4}{5} \left(c^{(8)} - \frac{1}{9} c^{(27)} \right) T_3 - \frac{2}{5} \left(c^{(8)} - \frac{1}{9} c^{(27)} \right) \left(T_3^2 + \frac{1}{4} \right) + c^{(1)}, \quad (12)$$

and for the baryon decuplet

$$\begin{aligned} M_{\Delta}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} + \frac{8}{63} c^{(27)} \right) T_3 + \frac{5}{63} c^{(27)} T_3^2 + \frac{1}{8} \left(c^{(8)} - \frac{2}{3} c^{(27)} \right) + c^{(1)}, \\ M_{\Sigma^*}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} - \frac{4}{21} c^{(27)} \right) T_3 + \frac{5}{63} c^{(27)} (T_3^2 - 1) + c^{(1)}, \\ M_{\Xi^*}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} - \frac{32}{63} c^{(27)} \right) T_3 - \frac{1}{4} \left(c^{(8)} + \frac{8}{63} c^{(27)} \right) \left(T_3^2 + \frac{1}{4} \right) + c^{(1)}, \\ M_{\Omega^-}^{\text{EM}} &= -\frac{1}{4} \left(c^{(8)} - \frac{4}{21} c^{(27)} \right) + c^{(1)}, \end{aligned} \quad (13)$$

and for the baryon antidecuplet

$$\begin{aligned} M_{\Theta^+}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} - \frac{4}{21} c^{(27)} \right) + c^{(1)}, \\ M_{N^*}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} - \frac{32}{63} c^{(27)} \right) T_3 + \frac{1}{4} \left(c^{(8)} + \frac{8}{63} c^{(27)} \right) \left(T_3^2 + \frac{1}{4} \right) + c^{(1)}, \\ M_{\Sigma_{10}^*}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} - \frac{4}{21} c^{(27)} \right) T_3 - \frac{5}{63} c^{(27)} (T_3^2 - 1) + c^{(1)}, \\ M_{\Xi_{3/2}^*}^{\text{EM}} &= \frac{1}{4} \left(c^{(8)} + \frac{8}{63} c^{(27)} \right) T_3 - \frac{5}{63} c^{(27)} T_3^2 - \frac{1}{8} \left(c^{(8)} - \frac{2}{3} c^{(27)} \right) + c^{(1)}, \end{aligned} \quad (14)$$

respectively.

Since the center of baryon masses absorb the singlet contributions to the EM masses with $c^{(1)}$, we safely neglect them for EM mass differences. At any rate, they are not pertinent to the EM mass differences in which they are canceled out. Therefore, the expressions of EM mass differences of SU(3) baryons have only two unknown parameters, i.e., $c^{(8)}$ and $c^{(27)}$, which were found to be

$$c^{(8)} = -0.15 \pm 0.23, \quad c^{(27)} = 8.62 \pm 2.39, \quad (15)$$

in units of MeV [41].

With SU(3) symmetry and isospin symmetry breakings considered, the masses of the SU(3) baryons can be expressed in terms of all the contributions discussed above

$$M_B = \overline{M}_B + M_B^{(1)} + M_B^{(2)} + M_B^{\text{EM}}. \quad (16)$$

III. RESULTS AND DISCUSSION

The mass splittings of the SU(3) baryons with SU(3) and isospin symmetry breakings to the first order were already investigated in Ref. [42]. Thus, we will show in this Section how to analyze the mass splittings of SU(3) baryons, considering the second-order corrections of isospin and SU(3) flavor symmetry breakings. Though the second-order effects of isospin symmetry breaking are rather small, we will take into account those effects, first for a consistency reason, and second for their practical importance. We will soon see that the results are more consistent with the existing experimental data. Though the second-order corrections of SU(3) flavor symmetry breaking to the mass splittings of the SU(3) baryon were already studied in Ref. [34], it is still incomplete, because it is not possible to use the baryon octet masses as input without isospin symmetry breaking. Furthermore, those effects of isospin symmetry breaking come into play in improving the mass splittings within the isospin multiplets.

The general method for determining all model parameters is very similar to the case of the first-order analysis [42]. We employ the least-square fit to adjust the model parameters, using as input the masses of the whole baryon octet, Ω^- , and Θ^+ from the LEPS experiment. In order to determine the masses of the decuplet and antidecuplet, we have to know at least the mass of one member in each representation. We have selected Ω^- and Θ^+ , because they are the isospin singlet in the decuplet and antidecuplet representations, respectively such that we can fix the center masses uniquely. One could choose other sets of baryons but they would yield the results that are phenomenologically worse.

The results of the fixed parameters are listed in Table I in comparison with those from the analysis with the first-order corrections only. As shown in Table I, the parameters with the second-order corrections are changed from those

Table I: The comparison of important parameters from the 1st order mass corrections with those from the full (to 2nd-order) ones. Input values of octet and decuplet baryon masses are taken from experimental data [48].

	First-order results	Full results
I_2	0.420 ± 0.006 fm	0.431 ± 0.001 fm
$m_{\text{ibr}\alpha}$	-4.390 ± 0.004 MeV	-6.458 ± 0.004 MeV
$m_{\text{ibr}\beta}$	-2.411 ± 0.001 MeV	-2.972 ± 0.001 MeV
$m_{\text{ibr}\gamma}$	-1.740 ± 0.006 MeV	-2.288 ± 0.008 MeV
$m_{\text{sbr}\alpha}$	-255.029 ± 5.821 MeV	-280.8 ± 14.2 MeV
$m_{\text{sbr}\beta}$	-140.040 ± 3.195 MeV	-129.3 ± 6.5 MeV
$m_{\text{sbr}\gamma}$	-101.081 ± 2.332 MeV	-99.5 ± 5.0 MeV
$\Sigma_{\pi N}$	36.4 ± 3.9 MeV	50.5 ± 5.4 MeV
c_{10}	0.0434 ± 0.0006	0.0482 ± 0.0021
c_{27}	0.0203 ± 0.0003	0.0231 ± 0.0012
a_{27}	0.0903 ± 0.0013	0.0994 ± 0.0004
a_{35}	0.0181 ± 0.0003	0.0210 ± 0.0013
d_8	-0.0434 ± 0.0006	-0.0482 ± 0.0021
d_{27}	0.0365 ± 0.0005	0.0450 ± 0.0042
d_{35}	0.1447 ± 0.0021	0.1625 ± 0.0078

with the first-order ones. Almost all parameters are altered by about $(20 - 30)\%$. In particular, the πN sigma term turns out to be (50.5 ± 5.4) MeV, which is almost 30 % larger than that with the first-order corrections ((36.4 ± 3.9) MeV). This can be easily understood from Eq. (3) in which the πN sigma term is expressed as $\alpha + \beta$. In fact, this sum of α and γ is enhanced in magnitude with the second-order corrections. Consequently, the πN sigma term is increased by about 30 % with the second-order corrections taken into account. In Ref. [2], $\Sigma_{\pi N} = 45$ MeV was used [49], while Ref. [34] obtained $\Sigma_{\pi N} = 73$ MeV in studying the baryon antidecuplet [50, 51]. It was discussed in Ref. [51] that larger values of the $\Sigma_{\pi N}$ are preferable to describe the then mass splitting in the baryon antidecuplet, because the debatable NA49 data of the $\Xi_{3/2}$ mass [53] was used for determining $\Sigma_{\pi N}$. Indeed, the larger value of the $\Sigma_{\pi N}$ reduces the antidecuplet splitting noticeably [31]. In Ref. [51], the $\Sigma_{\pi N}$ has been extracted by using the Θ^+ and $\Xi_{3/2}$ masses, based on the χ QSM: $\Sigma_{\pi N} = (74 \pm 12)$ MeV, which is quite larger than the present value. As we will show later, the predicted mass of $\Xi_{3/2}$ is larger than the NA49 data.

We are now in a position to present the main results of the present work, i.e., the masses of the SU(3) baryons. In Table II we list the reproduced masses of the baryon octet. The results indicate the stability of the numerical analysis. The sixth and seventh columns in Table II represent the reproduced octet masses respectively with the first-order and full contributions of isospin and SU(3) flavor symmetry breakings taken into account. The results with the full contributions are shown mostly to be closer to the input masses, as expected.

Table II: Reproduced masses of the baryon octet. The experimental data of octet are taken from the Particle Data Group (PDG) [48].

Mass [MeV]	T_3	Y	Exp. [input]	M_{B_8} (to 1st order)	M_{B_8} (full)
M_N	p	$1/2$	938.27203 ± 0.00008	939.8 ± 3.7	938.2 ± 8.5
	n	$-1/2$	939.56536 ± 0.00008	940.3 ± 3.6	940.3 ± 8.5
M_Λ	Λ	0	1115.683 ± 0.006	1109.6 ± 0.7	1110.2 ± 2.1
	Σ^+	1	1189.37 ± 0.07	1188.8 ± 0.7	1188.1 ± 0.8
M_Σ	Σ^0	0	1192.642 ± 0.024	1190.2 ± 0.8	1190.6 ± 0.9
	Σ^-	-1	1197.449 ± 0.030	1195.5 ± 0.7	1196.9 ± 0.8
M_Ξ	Ξ^0	$1/2$	1314.83 ± 0.20	1319.3 ± 3.4	1318.1 ± 7.1
	Ξ^-	$-1/2$	1321.31 ± 0.13	1324.5 ± 3.4	1324.8 ± 7.1

In Table III, we list the results of the masses of the baryon decuplet. The last column represents the final results of the present work with the full contributions considered. As shown in Table III, they are in better agreement with the experimental data in general, compared to those with the first-order corrections only (in the fifth column). The masses of the baryon decuplet are in general well reproduced.

Table IV presents the results of the masses of the baryon antidecuplet. Note that the mass of the Θ^+ is used as input. Since there are not enough experimental data, we can only compare the results of $N^*(1685)$ with the experimental data and find that they are in agreement with the data. The nature of this $N^*(1685)$ resonance is not reached yet in consensus. For example, Ref. [52] suggests that this resonance arises from coupled channel effects of the $S_{11}(1535)$, $S_{11}(1650)$, and $P_{11}(1710)$ resonances. However, the present analysis identifies it preferably as a member of the baryon antidecuplet.

Table III: The results of the masses of the baryon decuplet. The experimental data are taken from the Particle Data Group (PDG) [48].

Mass [MeV]	T_3	Y	Experiment [48]	$M_{B_{10}}$ (to 1st order)	$M_{B_{10}}$ (full)
M_Δ	Δ^{++}	3/2		1248.5 ± 3.4	1235.4 ± 8.0
	Δ^+	1/2	1231 – 1233	1249.4 ± 3.4	1236.8 ± 8.0
	Δ^0	-1/2		1251.5 ± 3.4	1239.7 ± 8.0
	Δ^-	-3/2		1255.1 ± 3.4	1243.9 ± 8.0
M_{Σ^*}	Σ^{*+}	1	1382.8 ± 0.4	1388.5 ± 0.3	1383.8 ± 1.7
	Σ^{*0}	0	1383.7 ± 1.0	1390.7 ± 0.4	1386.6 ± 1.7
	Σ^{*-}	-1	1387.2 ± 0.5	1394.2 ± 0.3	1390.8 ± 1.7
M_{Ξ^*}	Ξ^{*0}	1/2	1531.80 ± 0.32	1529.8 ± 3.4	1529.5 ± 6.8
	Ξ^{*-}	-1/2	1535.0 ± 0.6	1533.3 ± 3.4	1533.7 ± 6.8
M_{Ω^-}	Ω^-	0	1672.45 ± 0.29	input	input

Table IV: The results of the masses of the baryon antidecuplet.

Mass	T_3	Y	Experiment	$M_{B_{\overline{10}}}$ (to 1st order)	$M_{B_{\overline{10}}}$ (full)
M_{Θ^+}	Θ^+	0	$1524 \pm 0.005[15]$	input	input
M_{N^*}	p^*	1/2	$1685 \pm 0.012[22]$	1688.2 ± 10.5	1687.4 ± 6.8
	n^*	-1/2		1692.2 ± 10.5	1692.2 ± 6.8
$M_{\Sigma_{\overline{10}}}$	$\Sigma_{\overline{10}}^+$	1		1852.4 ± 10.0	1844.0 ± 0.2
	$\Sigma_{\overline{10}}^0$	0	0	1856.3 ± 10.0	1848.7 ± 0.3
	$\Sigma_{\overline{10}}^-$	-1		1859.0 ± 10.0	1852.1 ± 0.2
$M_{\Xi_{3/2}}$	$\Xi_{3/2}^+$	3/2		2016.5 ± 10.5	1993.7 ± 6.7
	$\Xi_{3/2}^0$	1/2		2020.5 ± 10.5	1998.5 ± 6.7
	$\Xi_{3/2}^-$	-1/2	-1	2023.1 ± 10.5	2001.9 ± 6.7
	$\Xi_{3/2}^{--}$	-3/2		2024.4 ± 10.5	2003.9 ± 6.7

As mentioned previously, it is known that the larger value of the $\Sigma_{\pi N}$ reduces the antidecuplet splitting noticeably [31, 34, 51]. However, the present result of $\Sigma_{\pi N}$ turns out to be smaller than that found in Ref. [34], even though we have considered the second-order contributions of isospin and SU(3) flavor symmetry breakings. Moreover, the mass splittings of the baryon antidecuplet remain rather stable with the second-order corrections. As discussed before, the sigma πN term is proportional to $m_{\text{ibr}}\sigma$ in Eq. (3). While the change of $m_{\text{ibr}}\sigma$ affects $\Sigma_{\pi N}$, the isospin mass splittings are rather insensitive to the second-order corrections of isospin symmetry breaking. As a result, the mass splittings of the baryon antidecuplet remain stable. Note that the masses of $\Xi_{3/2}$ are found to be larger than the NA49 data [53], though its existence is under debate. In Ref. [54] the mass ranges of the $\Sigma_{\overline{10}}$ and $\Xi_{3/2}$ were derived as $1795 < \Sigma_{\overline{10}} < 1830$ MeV and $1900 < \Xi_{3/2} < 1970$ MeV. The present results turn out to be slightly larger than those of Ref. [54].

Finally, it is interesting to mention that Ref. [44] defined a quantity measuring the strength of the second-order corrections of SU(3) symmetry breaking as

$$T = M_{\Xi^{*-}} - \frac{1}{2}(M_{\Sigma^{*-}} + M_{\Omega^-}), \quad (17)$$

which vanishes at the level of the first-order corrections, as pointed out in Ref. [44]. It is indeed so, since it becomes nonzero only when the second-order corrections are considered as follows:

$$T = m_{\text{sbr}}^2 I_2 \left[\frac{1}{128} \left(\alpha + \frac{5\gamma}{6} \right)^2 - \frac{5}{2688} \left(\alpha - \frac{\gamma}{2} \right)^2 \right] = (2.04 \pm 0.16) \text{ MeV}. \quad (18)$$

The present value of T seems smaller than the experimental one $(5.2 \pm 1.3) \text{ MeV}$.

IV. SUMMARY AND CONCLUSION

In the present work, we have investigated the masses of the SU(3) baryons within the framework of a chiral soliton model, taking into account SU(3) and isospin symmetry breakings to the second order in the perturbative expansion of the current quark masses. We also have considered the electromagnetic self-interactions that contribute to the isospin

mass splittings. In order to determine the unknown model parameters α , β , and γ , we employed the experimental data of the baryon octet, the Ω^- , and the Θ^+ . We then performed the minimization of the χ^2 . The second moment of inertia I_2 was also found, which is a key parameter to explain the mass splittings within the baryon antidecuplet. Moreover, the pion-nucleon sigma term was determined to be $\Sigma_{\pi N} = (50.5 \pm 5.4)$ MeV. The present results of the baryon decuplet masses are in remarkable agreement with the experimental data. The masses of $N^*(1685)$ in the antidecuplet turned out to be very close to the recent experimental data.

The present work is distinguished from the previous studies [2, 34] based on the chiral soliton model, which also deal with the mass splittings of the SU(3) baryons. The second moment of inertia I_2 plays a crucial role in explaining the heavier masses of the baryon antidecuplet, compared to those of the octet and decuplet. However, it was not possible to fix it unambiguously in previous works. In particular, since the $\Sigma_{\pi N}$ was not uniquely known empirically, some ambiguities were inevitable in previous analyses. While Refs. [2, 34] used the experimental data for the baryon octet, they did not consider isospin symmetry breaking, so that they were unable to incorporate whole experimental information. On the other hand, we were able to fix all model parameters by using the experimental data for the masses of the baryon octet, Ω^- , and Θ^+ , because effects of isospin symmetry breaking (both hadronic and electromagnetic parts) have been fully taken into account. Thus, we have produced the masses of the baryon antidecuplet as well as of the decuplet without any further adjustable parameter.

The vector and axial-vector properties of the SU(3) baryons can be investigated in a similar “model-independent” analysis. However, the previous analyses also suffer from ambiguities in determining parameters [31, 34, 55–57]. The parameters fixed within this work can be used in determining the magnetic moments and axial-vector constants of the SU(3) baryons. The related works are in progress [58].

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